

The goal of this project is to understand the beautiful combinatorial structure on the intersection theory of the moduli space of smooth curves imposed by Virasoro constraints. Via localization on the moduli space of stable relative maps (joint with T. Graber), the intersection theory of the moduli space of (smooth, pointed) curves may be connected with genus 0 “double Hurwitz numbers,” combinatorial objects connected with geometry, integrable systems, and the symmetric group. This connection implies an attractive structure of “top intersections” on the (non-compact) moduli space of curves. For example, this structure implies Faber’s intersection number conjecture for arbitrary genus and for up to three (and hopefully more!) points. (Faber’s intersection number conjecture is a theorem, but only by indirect means: via Givental’s proof of the Virasoro conjecture for the projective plane, by an argument of Getzler and Pandharipande. A satisfactory explanation of the combinatorial terms arising in the conjecture is still desirable.) This is an ongoing project, with I. P. Goulden and D. M. Jackson.