

Loop Algebras, Indecomposable Parabolic Bundles and Applications to Linear Algebra I, II, III

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The close relationship between Kac-Moody Lie algebras (and their quantizations) and representations of quivers is well-known, and it has proved to be very important, for example in the construction of canonical bases. Following work of Schiffmann, Lin and Peng, and others, one knows that there is a similar relationship between loop algebras of Kac-Moody Lie algebras and coherent sheaves on weighted projective lines (in the sense of Geigle and Lenzing), or equivalently, parabolic bundles on the projective line. I aim to explain this relationship, and to outline a proof that the dimension vectors of indecomposable coherent sheaves are exactly the positive roots for the loop algebra. I shall also explain how these ideas may be used to solve problems in linear algebra involving sums and products of matrices in prescribed conjugacy classes, including a partial solution of the Deligne-Simpson problem. It seems rather surprising that root systems of infinite-dimensional Lie algebras play a role here.